A simple but efficient concept of blended teaching of mathematics for engineering students during the COVID-19 pandemic

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Abstract

In this article we present a simple but efficient technological and logistic concept for the realization of blended teaching of mathematics and its applications in theoretical mechanics that was conceived, tested and implemented at the Department of Civil, Environmental and Mechanical Engineering (DICAM) of the University of Trento, Italy, during the COVID-19 pandemic. The concept foresees *traditional* blackboard lectures with a reduced number of students physically present in the lecture hall, while the same lectures are simultaneously made available to the remaining students, who cannot be present, via high quality low-bandwidth *online streaming*.

Based on our *first key assumption* that traditional blackboard lectures, including the gestures and the facial expressions of the professor, are even nowadays still a very efficient and highly appreciated means of teaching mathematics at the university, this paper deliberately does *not* want to propose a novel pedagogical concept of how to teach mathematics at the undergraduate level, but rather presents a *technical concept* of how to *preserve* the quality of traditional blackboard lectures even during the COVID-19 pandemic and how to make them *available* to the students at home via online streaming with adequate audio and video quality even at low internet bandwidth. The *second key assumption* of this paper is that the teaching of mathematics is a dynamic *creative process* that *requires* the *physical presence* of students in the lecture hall as audience so that the professor can instantaneously fine-tune the evolution of the lecture according to his/her perception of the level of attention and the facial expressions of the students. The *third key assumption* of this paper is that students need to have the possibility to interact with each other personally, especially in the first years at the university.

We report on the necessary hardware, software and logistics, as well as on the perception of the proposed blended lectures by undergraduate students from civil and environmental engineering at the University of Trento, Italy, compared to traditional lectures and also compared to the pure online lectures that were needed as emergency measure at the beginning of the COVID-19 pandemic.

Keywords: simple technical concept for blended teaching; transmission of traditional blackboard lectures of mathematics via high-quality/low-bandwidth audio & video streaming; bidirectional communication; blended chalk talks

1 Introduction and context of this work

The context of this work is the global COVID-19 pandemic, during which many governments worldwide have imposed severe restrictions of all kinds of activities, including a complete shutdown of traditional lectures at schools and universities which affected up to 83% of the total enrolled learners in the world, i.e. almost 1.5 billion people [22]. At the University of Trento, Italy, regular lectures completely ceased on March 5$^{\text{th}}$ 2020 by Decree of the Rector, just after the beginning of the second semester (end of February 2020 to mid June 2020) of the academic year 2019/2020. As a consequence, all teaching activities were entirely shifted to *online teaching*, which was a common choice of many universities worldwide [4].

In view of the favorable pandemic development in Italy during the summer months of 2020, at the University of Trento teaching activities with students present in the lecture hall were permitted again for the first semester of the academic year 2020/2021 (mid September 2020 to mid December 2020). However, due to national and regional COVID-19 restrictions, the number of students allowed to enter each lecture hall was *reduced* to 50% of its nominal capacity, see the Decree of the Italian Prime Minister of August 7$^{\text{th}}$ [6]. This restriction required the introduction of a suitable concept of *blended teaching*, where parts of the students are *physically present* in the lecture hall and the others are enabled to follow the lectures *online at home* via streaming over the internet. The blended teaching concept proposed in this paper has a particular focus on those lectures which in pre-COVID times were traditionally held at the blackboard and which were usually highly appreciated by the students in the past.

The ideas outlined in this paper are therefore based on a *first key assumption* that traditional blackboard lectures or chalk talks, which include the gestures and the facial expressions of the professor, are even nowadays still a very efficient means of teaching mathematics at the university level [10, 1, 16, 24]. The *second key assumption* of this paper is that the teaching of mathematics is a dynamic and *creative process* that *requires* the *physical presence* of students in the lecture hall as audience so that the professor can instantaneously fine-tune the evolution of the lecture according to his/her perception of the level of attention and the facial expressions of the students, in addition to the questions asked and the comments made by the students. The situation is similar to the one of an actor performing live in a theater in front of a big audience [2]. The authors are convinced that this particular relationship between the professor and the audience not only improves the performance and the quality of the teaching for the audience present in the lecture hall, but also the quality for those who follow the lectures online. In the opinion of the authors, a similar emotional experience is completely impossible when a professor teaches a pure online lecture merely in front of a computer screen, without any possibility to directly see or perceive the emotions of his/her audience. The *third key assumption* of this paper is that students need to have the possibility to *interact* with each other personally [18], especially in the first years at the university when students should have an opportunity to get to know each other and to *socialize* [25], although this seems to be a contradiction to the social distancing measures adopted during the COVID-19 pandemic. Allowing the students to physically come to the lecture halls in order to live the lectures and exercises together as a common experience, despite the anti-COVID rules, seems to be still more attractive than following pure online lectures alone in front of a computer screen.

As a consequence, our concept aims at enabling those students who can follow the lectures only online to have an experience that is as close to a real classroom experience as possible, permitting the students online to communicate also directly with the professor and allowing them to have a high quality view of the professor and the blackboard, combined with high audio quality. These are all together nontrivial requirements, in particular due to pandemic *internet bandwidth* restrictions. According to our three key assumptions, this paper therefore *deliberately* does *not* present a novel pedagogical concept of how to teach mathematics at the university, but on the contrary, aims at showing a simple but efficient *technical concept* of how to *preserve* the quality of traditional means of teaching mathematics at the university level in the context of the COVID pandemics as far as possible.

Innovative concepts of blended teaching in pre-COVID times were discussed in [12, 23], while recent contributions in the field [9, 8, 11, 5, 17, 13] also have a particular focus on the use of blended teaching concepts during the COVID-19 pandemic.

The rest of this paper is organized as follows: in Section 2 we present and motivate the concept of blended teaching developed and implemented at DICAM during the first semester of the academic year 2020/2021. The motivation of our blended teaching concept is also made in the light of the *pure online* teaching that was adopted as compulsory *emergency measure* during almost the entire second semester of the academic year 2019/2020. In Section 3 we describe the details of the technological and logistic realization of the concept, in particular also the hardware and software requirements, the special training of the professors before the start of the semester and the organization of the technical support for blended teaching that was made available during the semester. The quality of the adopted concept was quantitatively evaluated by both, professors and students via online questionnaires. The results of this evaluation are presented and discussed in Section 4. The paper closes with some concluding remarks in Section 5. The electronic supplementary material of the paper contains some screenshots of how the lectures were perceived by students who followed the lectures online.

2 Blended teaching concept at DICAM during the COVID-19 pandemic

In order to illustrate and to motivate the blended teaching concept elaborated and implemented by DICAM during the first semester of the academic year 2020/2021 (Sections 2.3-2.4), we first briefly describe the typical structure of the courses in pre-COVID times (Section 2.1) and also draw some conclusions from the pure online teaching phase that was mandatory in spring 2020 at the beginning of the pandemic (Section 2.2).

Table 1: Survey of DICAM made among its students in June 2020 at the end of the emergency online teaching of the second semester of 2019/2020. Total number of students participating in the survey: 445. Not all questions were mandatory.

Questions	Answers with absolute numbers
1. Which online teaching modality do you prefer?	asynchronous (47)
	synchronous without registration (17)
	synchronous with registration (257)
	depends on the lecture (67)
	depends on the professor (42)
	no opinion (15)
2. Seeing the body language and the facial expressions of the professor makes the lecture more interesting?	0 - no opinion (20)
	1 - not at all (1)
	2 - a bit (17)
	3 - a lot (87)
3. The use of the blackboard as a traditional means of teaching is useful?	0 - no opinion (29)
	1 - not at all (1)
	2 - a bit (11)
	3 - a lot (81)
4. Do you consider the option to attend the lectures of the first semester of 2020/2021 only online?	**0 - no (263)**
	1 - yes (162)

2.1 Typical structure of the courses in mathematics offered at DICAM

All courses at DICAM in the areas of mathematics and its applications, including theoretical fluid mechanics and solid mechanics, which are under consideration in this study, follow the traditional scheme of theoretical lectures combined with classroom exercises. In addition, for courses offered at the bachelor level, there are special group exercises organized in small work groups, led by MSc or PhD students (tutors). The group exercises are based on the elaboration and discussion of exercise sheets in small work groups and make use of a flipped-class concept, where students first elaborate the exercises at home and then discuss the results with the tutors in the classroom [14, 7]. Online surveys and structured interviews with the student representatives held in pre-COVID times revealed a clear preference of the students of DICAM for traditional blackboard lectures, documented in the *Yearly report of the joint committee of professors and students of DICAM*, which is not available for public view.

2.2 Experience of the emergency phase during the second semester 2019/2020

With the shutdown of regular lectures in March 2020, all teaching activities at DICAM suddenly needed to be held entirely online. In addition to the combined online teaching platform Moodle plus Kaltura[1], which was already available in pre-COVID times, for the emergency online teaching the University of Trento also provided its professors with licenses for the software ZOOM[2]. Online teaching could be held either in a *synchronous manner* by direct online streaming of the lecture content via ZOOM, or in an *asynchronous way* by pre-registering the lectures and uploading them into the combined Kaltura-Moodle platform. The personal experience of the second author of this paper made with the pure online teaching activities from March to June 2020 was overall rather negative, mainly due to the complete lack of an instantaneous feedback from the students (missing facial expressions, comments and questions) even during synchronous teaching, since all students kept their microphones and webcams systematically switched off in order to save internet bandwidth, but also due to the lack of the blackboard as a traditional means of teaching mathematics at the university. In a systematic survey made by the Department in June 2020, at the end of the emergency online teaching during the second semester 2019/2020, and to which 445 students of DICAM responded, the *clear preference* of the students for online teaching was the *synchronous online lecture* with the registration of the lecture *made available* to the students via the Moodle-Kaltura platform afterwards. In the same online survey the students also expressed a *clear preference* for *traditional lectures at the blackboard*, confirming that the body-language of the professor is important and that the possibility to see the professor renders the lecture more interesting. For a summary of the main results of this survey, see Table 1, from which it also becomes evident that the majority of the students did not consider the option to attend lectures only online in the case a traditional lecture would again be possible in the subsequent semester.

[1] `https://moodle.org` and `https://corp.kaltura.com/`
[2] `https://zoom.us`

Based on the results of Table 1 and in order to overcome the shortcomings of the purely online emergency teaching of the second semester 2019/2020 and taking also into account the updated national and regional regulations that permit regular teaching activities under certain conditions, the Department of Civil, Environmental and Mechanical Engineering (DICAM) has adopted the following strategy for the first semester of the academic year 2020/2021 detailed in the next section.

2.3 Main pillars and general objectives of the blended teaching concept

Given the clear preference expressed by the students of DICAM in favor of *traditional blackboard lectures* actually held in the lecture hall and given also the reciprocal importance of the body language and the facial expressions of the professor seen by the students and, vice versa, the ones of the students seen by the professor, the Department has therefore decided to adopt a simple blended teaching concept which tries to create an environment that is *as normal as possible*, both, for the professors and for the students.

The concept consists in the *high quality transmission* of traditional blackboard lectures that are actually held in the lecture halls in front of up to 50% of the students who are allowed to be physically present and to make this experience *available* as much as possible via *low-bandwidth online streaming* to the remaining students who have to follow the lectures online at home due to the COVID-19 restrictions. The two most important pillars on which our blended teaching concept relies are the *high quality view* of the blackboard and the professor, combined with *high audio quality* to capture the speech of the professor, not only for the students present in the lecture hall, but also for those online. Under pandemic internet bandwidth restrictions, both pillars were anything else than trivial to realize, but in the next section we will give technical details of how these objectives can be reached with standard hardware and software. Furthermore, *bidirectional communication* is integral part of the blended teaching concept discussed in this paper, allowing thus the students who follow the lectures online to communicate *directly* with the professor, and making their contributions, questions and comments also available to all the students who are physically present in the lecture hall. As already stated in the introduction, the blended teaching concept adopted by DICAM *deliberately* does *not* present a novel *pedagogical concept* of how to teach mathematics, but on the contrary, aims at showing a simple but efficient *technical concept* of how to *preserve* the quality of traditional means of teaching mathematics in the context of the COVID pandemics as far as possible. The three main objectives were: i) to *preserve* the high quality of traditional blackboard lectures which were usually appreciated by the students in the past pre-COVID era also during the COVID pandemic and to make this form of lecture also *accessible* to all students who were not able to attend the lectures in person, in particular also to those with *low internet bandwidth*; ii) to allow students to interact personally with each other and with the professors, complying with all the restrictions due to the COVID pandemic like maintaining a minimum distance between each other and the obligation to wear a face mask during the entire period in which the students are present in the building; iii) to allow professors to continue using their well-established traditional teaching concepts as far as possible, even under pandemic conditions. The same blended teaching concept was also adopted for the group exercises led by the tutors, allowing small work groups of students to attend the group exercises in person, while the others were following online.

The obvious *shortcomings* of the blended teaching concept illustrated in this section are the rather stringent requirements on the computer hardware and software to be used in the lecture hall, combined with the necessary technological skills that needed to be acquired by the professors before and at the beginning of the semester.

To realize the concept detailed above, *all* lecture halls of the Department needed to be technically upgraded with an appropriate audio and video system, since no special equipment for blended teaching was present at DICAM in the pre-COVID era. The Department furthermore organized a series of special training sessions, in which the professors were trained to use the necessary hardware and software in order to get ready for blended teaching before the beginning of the semester. In order to allow the students to get a preview of how the blended teaching would be in the first semester of 2020/2021 and in order to test the equipment and to show professors the technical possibilities, the students were invited to join some of these training sessions online.

2.4 COVID-specific part of the concept

In order to establish *a priori* which students were allowed to enter the lecture hall in a given week, and which were not, the Department has carried out an online survey before the beginning of the semester in which the students could express their intention whether they wanted to attend lectures in person, or not. The choice was *free*, but *binding*. As a result of this survey, in which only about 18% of all students chose to follow lectures exclusively online, the Department divided those students who wanted to attend lectures personally in sub-groups that would be allowed to come to the lecture halls based on a *weekly rotation principle* so that all students had the same number of weeks in which they could attend the lectures in person and in which they needed to follow the lectures online, respectively. A maximum nominal capacity of 50% of each lecture hall combined with the percentage of students who decided to attend all lectures only online allowed most of the students of DICAM to be actually present in the lecture halls for most of the weeks.

An important choice of the adopted concept was to allow each professor to use his/her own laptop for the blended teaching, for two reasons: i) using their own laptops, professors can use their preferred computer environment and operating system with which they have most experience and which they are most acquainted with; ii) using their own laptops avoids touching common computer keyboards and pointing devices and therefore solves the problem of disinfection of these devices between one lecture and the other.

To reduce the stress induced by the obligation of wearing face masks continuously while present in the University building and to reduce the pressure on the public transport system, the timetable was re-organized in a morning block (8:30-13:30) and an afternoon block (14:30-19:30), so that some students were present only in the morning block and others only in the afternoon block. As anti-COVID measure the Department allocated one and the same lecture hall only for at most two different groups of students, with an interval of one hour left between the two groups that was needed for cleaning purposes. Furthermore, one medium-size lecture hall was kept empty as strategic reserve in case of COVID-related disinfection measures or in case of technical difficulties in another lecture hall. The adopted timetable leaves a total amount of at most 25 hours of real lectures in the lecture hall per group, which is not always sufficient to fit the entire timetable of each group of students. To mitigate the effects of this decision, before the start of the semester, all professors of the Department could choose in a free but binding manner whether to offer their lectures purely online, or whether they wanted to adopt the blended concept outlined above. Most professors preferred the blended option for their courses (70%), but those 30% who chose the online teaching were enough to fit all blended courses into the given 25 hour limit. The pure online lectures were mostly held in synchronous manner and could be followed by the students *only at home* and *not* from the university building. For this reason, the pure online lectures were scheduled only at a 2 hours distance from blended lectures, in order to allow students to move from the university building to their homes, and vice versa.

The entrance and the exit of the university buildings were controlled electronically via a smartphone app that was specifically developed by the ICTS services of the University of Trento and which required the scanning of a QR code upon entry and exit of the university buildings. The students who were allowed to attend lectures in a given week were authorized top-down by an electronic authorization system that was coupled with the cellphone app. Queues at the entrance were reduced by slightly shifting the start of the lectures at the MSc level and those at the BSc level by 15 minutes.

3 Technological and logistic realization of the concept

To realize the concept described above, the personal laptop of each professor became the crucial hub for the acquisition and distribution of the audio and video streams to be sent online. As videoconferencing system we employed ZOOM, already licensed by the University of Trento during the first phase of the pandemic and due to its very good performance concerning audio and video live streaming even in the case of low available internet bandwidth. In order to minimize technical problems *a priori*, two completely *redundant* systems were installed in each lecture hall. A videocamera system that allows to transmit traditional blackboard lectures, but also *classroom experiments*, as well as a graphics tablet and digital pen system [26, 15]. In this manner, each professor was also offered the free choice of which means he/she wanted to use for teaching, eventually also a combination of both (videocamera + tablet). The videocamera system was also mandatory for those lectures which involve classroom experiments, very frequent in theoretical solid mechanics [3], see e.g.[3]. The resulting main *challenges* that needed to be overcome to implement this strategy were the following:

- The lecture halls at DICAM were not at all prepared for blended teaching and thus needed to be specifically equipped;
- Achieving sufficiently high video resolution online that allows to read complex mathematical formulas clearly, despite the existing COVID bandwidth and HD video feature restrictions;
- Achieving at the same time a sufficiently high audio quality both in the lecture hall and online, in particular capturing the speech with high quality while the professor is turned towards the blackboard while writing and allowing also a clear bidirectional communication;
- The fact that rather complex and relatively new technology needed to be handled by each professor during his/her blended lectures;

[3] http://www.ing.unitn.it/~bigoni/flutter.html

- The highly heterogeneous computer hardware and operating systems resulting from the choice of allowing each professor to use his/her personal laptop, which also made the training and technical support particularly challenging and widen the necessary adapters to connect to the standard hardware installed in each lecture hall.

In order to convey an idea of the scale at which we worked, let us also note that the concept presented in this paper was applied to the entire Department, covering more than 50 courses from the fields of mathematics, physics, chemistry, engineering and social sciences. In what follows, we will further comment the above challenges indicating the strategies employed to surmount them.

3.1 Description of the employed equipment

The implementation of the developed blended teaching concept required the lecture halls to be furnished with a completely new set of electronic devices. The initial equipment of the classroom accounted for a blackboard, the projector system and, in large lecture halls, also a sound system so that the professor can be clearly heard on site. On top of this we introduced:

- Personal laptop. As already mentioned, the particular setup designed gives the professor the opportunity to employ his/her own laptop aiming at minimizing the novelties yielding from the use of new technical devices. Consequently, the laptop of the professor became the main control device of the blended teaching lectures.
- Camcorder. To properly capture the blackboard and transmit physical experiments a full high definition camcorder was needed. It was further equipped with a directional microphone to record the speech of the professor in small classrooms. For large blackboards, instead of the standard camcorder proposed here, it was necessary to employ a wide angle lens camera (e.g. PTZ Minrray UV5C40) controlled by IR remote control to allow switching between different parts of the blackboard and different levels of zoom.
- Camlink 4K. The transmission of a high quality video signal from the camcorder to the computer required for a 4K HDMI to USB converter. Let us note that although the quality broadcast by these devices goes up to 1080p at 60 fps or 4K at 30 fps, to reduce bandwidth consumption we suggest to limit the video capturing program to 1080p at 25fps.
- Active USB 3.0 extension cable. Typically the camcorder and, consequently, the camlink are far away from the laptop so a USB 3.0 extension cable is needed. To properly preserve the video signal and provide the needed power supply for the camlink, an active cable was employed.
- USB Hub 3.0. To minimize the number of connectors to be plugged into the laptop, they were all gathered in an unique USB hub of four ports that transmits input and output signals to and from the computer. At least SuperSpeed USB (USB 3.0) cables, connectors and ports are needed to transfer all the signals, being preferred the use of USB 3.1 or 3.2 ports over USB 3.0 ports.
- Graphics tablet and digital pen system. The alternative system to the blackboard was a rubber grip HD 22 inch graphics tablet not hand sensible (i.e. a pen display), so that the writing is as close as possible to traditional handwriting on a piece of paper, used together with the preferred interactive whiteboard software. To be able to import and export pdf documents and save the lecture notes we have suggested the use of the OpenBoard[4] software. Communication of the graphics tablet with the computer was achieved via an input HDMI signal that duplicated the laptop screen and an output USB signal for the recording of the handwriting.
- HDMI splitter. The laptop screen must be shared both with the students in the lecture hall using the projector system and with the graphics tablet. To duplicate the signal an HDMI 4K splitter is employed. Since most projectors at DICAM are only equipped with a VGA input port a HDMI to VGA converter is connected to the HDMI cable exiting form the HDMI splitter.
- Airlink. To facilitate bidirectional communication the comments of online students were broadcast through the pre-existing sound system of the large lecture halls using an airlink connected to the laptop via bluetooth. For small and medium classrooms (up to 100 students) and without a pre-installed sound system an alternative was the use of a speakerphone (Jabra Speak 810) that can be connected to the laptop using the audio jack or via bluetooth. The latter device allows listening to the online comments in the lecture hall and captures also the voice of the people present in the classroom to be transmitted online, while the former device only allows the online students to be heard in the lecture hall.

[4] https://openboard.ch/index.en.html

Table 2: List of selected components employed in each lecture hall and corresponding cost estimate in EUR including 22% VAT. The equipment was carefully selected and tested before installation.

Equipment	Cost estimate in EUR
Laptop of the professor	0
Panasonic HC-V770 camcorder	408
Elgato Camlink 4k HDMI to USB converter	130
Ugreen 10m active USB 3.0 extension cable	43
Anker USB hub 3.0	13
Wacom Cintiq 22 graphics tablet	953
4k HDMI splitter Ablewe	16
Primewire 2m HDMI cable	10
Ugreen HDMI to VGA adapter	10
BT DW 20BR Klark Teknik bluetooth airlink	89
Jabra talk 25 bluetooth ear microphone*	28
Total amount:	**1700**

*one ear microphone was needed for each professor to comply with anti-COVID regulations

- Ear microphone. In large lecture halls and when the professor is turned towards the blackboard, the use of a directional microphone was not enough to properly capture the speech with high quality. To solve this problem a personal bluetooth ear microphone was employed by each professor.
- Adapters. The resulting hardware of each lecture room, as explained below, requires the personal laptop of each professor to be connected thorough one HDMI and one USB/A port. Due to the great heterogeneity of modern computer hardware, a large set of adapters has been made available allowing to use also VGA, DisplayPort, mini DisplayPort, mini HDMI, micro HDMI and USB-C ports.

A detailed list of the above components and their estimated cost can be found in Table 2.

3.2 Physical setup of the equipment

A sketch of the connections among the different devices is included in Figure 1. On the one hand, to allow high quality capturing of the blackboard, the full HD video camera is connected via HDMI cable to a camlink 4K which allows the transition to a USB 3.1 signal transmitted using an active USB extension cable to an USB hub connected to the laptop of the professor. The alternative and/or complementary device for the blackboard is the pen display which transfers the data collected via a USB 3.1 cable connected to the USB Hub. Meanwhile, the signal of the computer arrives to the tablet using an HDMI cable. To allow the display of the laptop screen both in the graphics tablet and in the classroom projectors an active HDMI splitter 4K is employed. Consequently, a single HDMI cable will take off from the laptop arriving at the HDMI splitter from where two different HDMI cables transmit the same signal to the tablet and the projector; if necessary a HDMI to VGA adapter is employed. Let us note that the HDMI splitter is also connected to the USB hub from which it receives the needed power supply. The speech of the professor is picked up from a bluetooth ear microphone and collected in the computer. To ease bidirectional communication the laptop is connected using a bluetooth airlink to the sound system of the lecture hall.

3.3 High quality audio and video online streaming

High quality low bandwidth video streaming via screen sharing. In order to share visual information, the lecture halls were equipped with full HD video cameras, that capture the blackboard content, and graphics tablets. In presence students can see the contents directly at the blackboard or on the projector screens. For online students the visual information were streamed via ZOOM. But ZOOM automatically reduces the video quality to 640×360p due to COVID-19 internet bandwidth restrictions for all meetings with three or more people simultaneously connected. This resolution was *far too low* to transmit the content of a blackboard with complex mathematical formulas, containing also small indices and superscripts, properly to the online students. The only option to overcome this intrinsic limitation and to obtain a proper transmission of the video signal of the blackboard in permanent full HD resolution (1920×1080p) was to use the **share screen feature** of ZOOM. While this was rather obvious in the case of the use of a graphics tablet, it required the use of a third party video capturing software to capture and transmit the signal coming from the video cameras installed in the lecture hall. We found the following free video capturing software

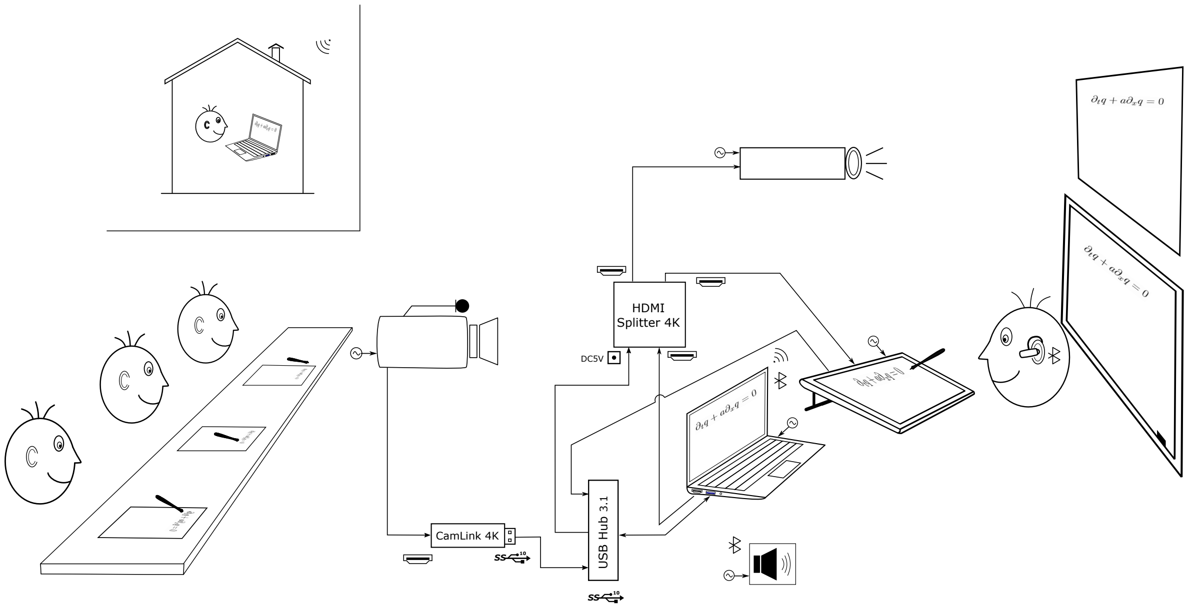


Figure 1: Sketch of the technical realization of the blended teaching concept at DICAM.

particularly useful: VLC and OBS[5] on Windows and Linux and Quicktime[6] on Macintosh computers. The reasonable *compromise* in this choice lies in the *reduced framerate* of the video, but maintaining the full HD resolution at all times, which is *crucial* to transmit the static content of the blackboard in optimal quality.

Audio streaming. The bluetooth ear microphone is *crucial* for blackboard lectures in order to suppress background noise and allowing the professor to talk while writing on the blackboard. This ear microphone, which serves to transfer the audio in high quality to the students attending the lecture online, is worn *in addition* to the standard classroom microphone that serves to amplify the audio signal in the lecture hall.

Bandwidth measurements. A series of experiments has shown that with our strategy of video streaming via screen sharing the average framerate of the video signal of a traditional blackboard lecture transmitted online via ZOOM was still about **24 fps** at permanent full HD resolution, while the total bandwidth required in download by the online students was on average only **700 kbps**, and hence fully adequate even for low bandwidth ADSL and average 4G connections. These results confirm that the adopted strategy is adequate for the high quality online streaming of mathematics lectures for most students with average internet connection.

Recorded lectures. As a backup solution and to mitigate internet connection problems and difficulties related to different time zones, most lectures were also recorded using ZOOM. They were then made available to the students using the online teaching platform Moodle plus Kaltura video cloud service. As it is reflected in the results of the questionnaires described in Section 4, the possibility of recording lectures may lead to an added value on the learning experience by allowing also the students attending in a synchronous way to review particular parts of the lectures when autonomously studying the subject.

3.4 Lecture setup

The former setup of the on site equipment has been designed aiming at minimizing the complexity of the connections from the professor point of view. When arriving in the lecture hall the laptop of the professor needs to be physically connected only with a single input/output USB/A cable (coming from the USB hub) and one HDMI cable (towards the HDMI splitter). Then, the sound system and the ear speaker are paired via bluetooth. Once all connections are established the ZOOM meeting is opened selecting the correct input and output sound devices and the *share screen* mode is activated. At this point the professor should select the methodology he/she would like to employ for the lecture:

I. Blackboard. In this case the auxiliary video capturing software is opened (VLC, OBS, Quicktime), the camcorder is selected as input source and the regular video stream of ZOOM must be turned off.

[5] https://www.videolan.org/ and https://obsproject.com/
[6] https://www.apple.com

II. Graphics tablet. OpenBoard or the preferred interactive whiteboard software is employed. The students in the lecture hall will be able to see the screen of the tablet on the projector screen.

III. Third party software. While sharing the screen any third party software like classical mathematical programs such as Matlab, Maple, R Studio, Mathematica, or PDF and Powerpoint files can be shown.

Since all three possibilities rely on *screen sharing* via ZOOM and all devices are simultaneously connected, changing between the different options becomes straightforward!

3.5 Training and technical support

Despite the lecture setup described above aimed at being as simple as possible from the professor point of view, the blended teaching concept may happen to be a complete novelty for the teaching and technical staff and for the students.

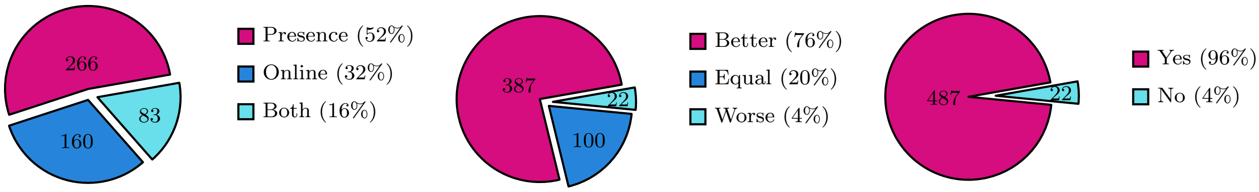
Thus, already *before the start* of the lectures, we organized a series of special *training sessions*, in which the professors were trained to use the necessary hardware and software in order to get ready for blended teaching before the beginning of the semester. During the first part of these sessions, the new equipment was presented via demonstrations of how each of the three possible lecture modalities (namely the use of blackboard, graphics tablet or third party software) was practically working. Moreover, in order to allow the students to get a preview of how the blended teaching would have been in the first semester of 2020/2021, they were also invited to join some of these training sessions remotely via ZOOM and to ask any questions they had. The presence of hundreds of online students during the demonstrations allowed to show and convince the students and professors of the effectiveness of the adopted concept, included the bidirectional communication. In the second part of the training sessions, we gave the possibility to each professor (in turn) to personally test the new equipment in order to really familiarize with it, verify the compatibility of their personal laptop with the standard hardware of the lecture halls and be aware of the eventually needed adapters. This was also an occasion for training the technical staff and a special group of students that was hired for adjoint help during the semester.

Indeed, to allow blended lectures to start and proceed smoothly, a *continuous technical support* was organized *throughout the entire semester*, operated by the technical staff of DICAM and by this group of specially trained students. During the first week, we offered a *top-down* supporting service, being a person of our staff present at the beginning of each lecture in each room ready to facilitate the lecture setup. During the rest of the semester, due to the high level of autonomy reached by each professor, the offered service was changed to the *bottom-up* or *on-demand* type, i.e. only in case of necessity the professor could request the intervention of the technical staff always present in the building.

4 Evaluation of the concept and of its practical implementation

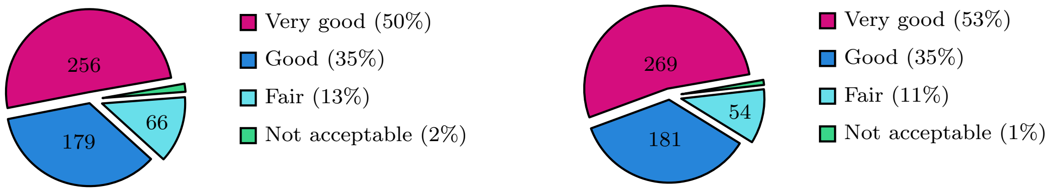
Here we present quantitative results of the evaluation of the blended teaching concept illustrated in the previous sections. Evaluation was carried out separately by the professors and the students via specific online questionnaires. The concept presented in this paper was applied to the entire DICAM department, covering more than 50 courses and 18 lecture halls. For a sample of interest in the context of this journal, we considered all the courses involving teaching of mathematics and its application which result also to be representative of each year of study, namely the course of *Analysis 1* (BSc, first year), *Analysis 2* (BSc, second year), *Theoretical solid mechanics* (BSc, third year), *Theoretical fluid mechanics* (BSc, third year) and *Numerical analysis for PDE* (MSc, first year). We received a total of 509 answers from the students, and we collect the opinions of the 6 professors leading the didactic team of each course. The data of the questionnaires was collected after one month of blended lectures.

More than half of the total answers comes from students attending the courses mostly in presence, a third of them attended mostly online and a part of them chose the preferred modality on a daily base depending on personal necessities. Almost the totality of the received answers shows a *general satisfaction* for the implemented blended teaching concept and a *clear preference* for blended teaching with respect to pure online teaching, see the detailed results reported in Figure 2. Indeed, the overall video and audio quality was very positively judged by around 85% of the students, see Figure 3 for more details about the answers. Moreover, the key assumptions guiding the conception of our blending teaching approach were confirmed by the opinions of DICAM students: see Figures 4-5 where we summarize the results of *check box*-type questionnaires asking which new features introduced with blended teaching were important for them. With respect to pure online teaching, students appear to appreciate the major and easier interaction with professor and colleagues and the increased effectiveness of lectures thanks to the use of the blackboard and the possibility to easily see professors body-language and expression. With respect to standard traditional



(a) Attended modality (b) Blended vs. online (c) General satisfaction

Figure 2: Questionnaire results on the opinion on blended teaching versus pure online teaching (b) and general satisfaction for our blended teaching implementation (c), from students attending in presence and/or online according to (a). Total answers received: 509. Absolute values inside the graphs, approximate percentage in the legend.



(a) Video quality (b) Audio quality

Figure 3: Questionnaire results on the video (a) and audio (b) quality of our blended teaching implementation. Total answers received: 509. Relevant absolute values inside the graphs, approximate percentage in the legend.

teaching, they appreciated the new high flexibility of switching between in presence and online lectures, not only for COVID related emergency reasons, but also for normal everyday life organization. The availability of recorded lectures was greatly appreciated also in the free comment space.

For what concern professors, they show a clear preference for the blended teaching concept with respect to the pure online teaching; in particular, they highly appreciated the possibility to continue using traditional blackboards, showing experiments and having a true interaction with the students physically present in the lecture hall. Some of them are also willing to maintain several of the concepts here introduced even when traditional teaching will be again possible, considering indeed that the possibility of recording, the increased flexibility and the new available technological instruments can provide an added value to the lectures and widen the reached target.

Moving beyond: innovative didactic concepts for the numerical analysis course. Given the available new technological equipment and in order both i) to mitigate the adjoint difficulties students can perceive due to the pandemic crisis, but also ii) to provide additional and high quality learning opportunities, a specific teaching concept was designed and tested for the course *Numerical analysis for PDE* (MSc, first year) held in 2020/2021 by the first two authors of this paper. For the contents of the course we refer to [19, 21]. In addition to the blended theoretical lectures at the blackboard, according to the technical concept outlined in this paper, the course offered: i) weekly synchronous online computer exercises with from scratch implementation of the studied numerical methods, ii) weekly exercise sheets with reference solutions provided under the form of a *podcast/screencast*, i.e. audio and video recordings of the commented solutions of the exercise sheets, iii) two projects concerning the numerical solution of simplified real life applications, to be developed in small groups and with students documenting their findings in a written report and presenting the methods and results in class, following the concepts of a *continuous assessment* of the students [20] and the *flipped classroom principle* [7, 5]; iv) mixed use of Italian and English language to also transmit specific technical vocabulary and the capacity to convey subject related concepts at international level.

After five weeks of lectures, corresponding to almost the 50% of the entire course, the 25 students answering the questionnaire were overall satisfied of this new offer, all of them found useful the weakly exercise sheets and the commented solutions in the form of podcast, and 22 students appreciated the mixed language approach with 18 of them thinking this results in an added value for the course.

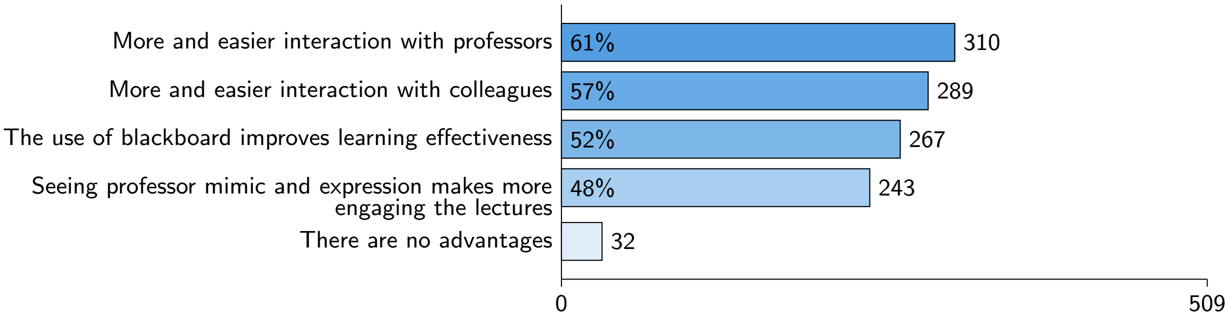


Figure 4: New features appreciated in blended teaching vs. pure online teaching. Total answers received for the *check box*-type questionnaire: 509. Absolute value at the end of bars, approximate relevant percentage inside bars.

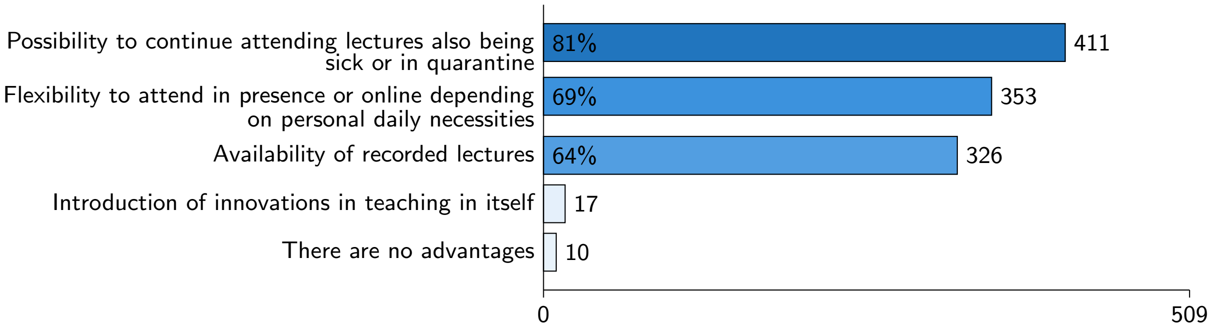


Figure 5: New features appreciated in blended teaching vs. traditional teaching. Total answers received for *check box*-type questionnaires: 509. Absolute value at the end of bars, approximate relevant percentage inside bars.

5 Conclusions

In this paper we have shown all the technical details to realize an economically affordable and simple but efficient technical concept for the realization of blended teaching of mathematics for engineering students during the COVID-19 pandemic. The concept is based on the three key assumptions i) that traditional blackboard lectures, including the gestures and facial expressions of the professor are still a very efficient and highly appreciated means of teaching mathematics at the university; ii) that a lecture in mathematics is a creative process that requires the physical presence of an audience so that the lecture can be best adjusted to the needs of the students and finally iii) that undergraduate students need a minimum level of direct personal interactions and the possibility to get to know each other, especially in the first years. All three basic assumptions, as well as the perception of the audio and video quality have been quantitatively verified at the aid of systematic online surveys that were sent to all students. The overall level of satisfaction with the blended teaching concept presented in this paper was about 96%. A series of direct measurements revealed that the total internet bandwidth required in download for full HD video streaming via screen sharing with ZOOM was rather low (only about 700 kbps) and thus makes the methodology applicable also during pandemic internet bandwidth restrictions and makes blended teaching also available for those students who have a fairly slow internet connection. If necessary and if possible, this concept could be adapted also in subsequent semesters.

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A Supplementary material

In this appendix we provide a comparison of the video quality perceived by the students online when using the concept suggested in this paper (full HD screensharing with third party video capturing software at 1080p) or the standard ZOOM video feature, which automatically reduces the video quality for meetings with more than two participants to 360p, due to pandemic internet bandwidth restrictions. The shown blackboard content deliberately contains complex mathematical formulas with small superscripts and indices. We also show a photo of the original blackboard content directly taken in the lecture hall, which serves as a reference. The screenshots were taken on an Asus Zenbook with full HD screen and ZOOM client 5.3.1 on Windows. For the video streaming via screen sharing we used ZOOM in combination with OBS Studio 25.0.8. The video signal was captured with a Sony AX-53 camcorder and an Elgato 4k camlink. From Figs. 6 and 7 we can clearly conclude that the concept of full HD video streaming via screen sharing proposed in this paper provides a significantly improved video quality compared to the standard ZOOM video feature. In Fig. 7 the blackboard content is clearly readable, while in Fig. 6 it is not. Fig. 8 shows a photograph directly taken in the lecture hall, which serves as a reference to assess the quality of Figures 6 and 7.

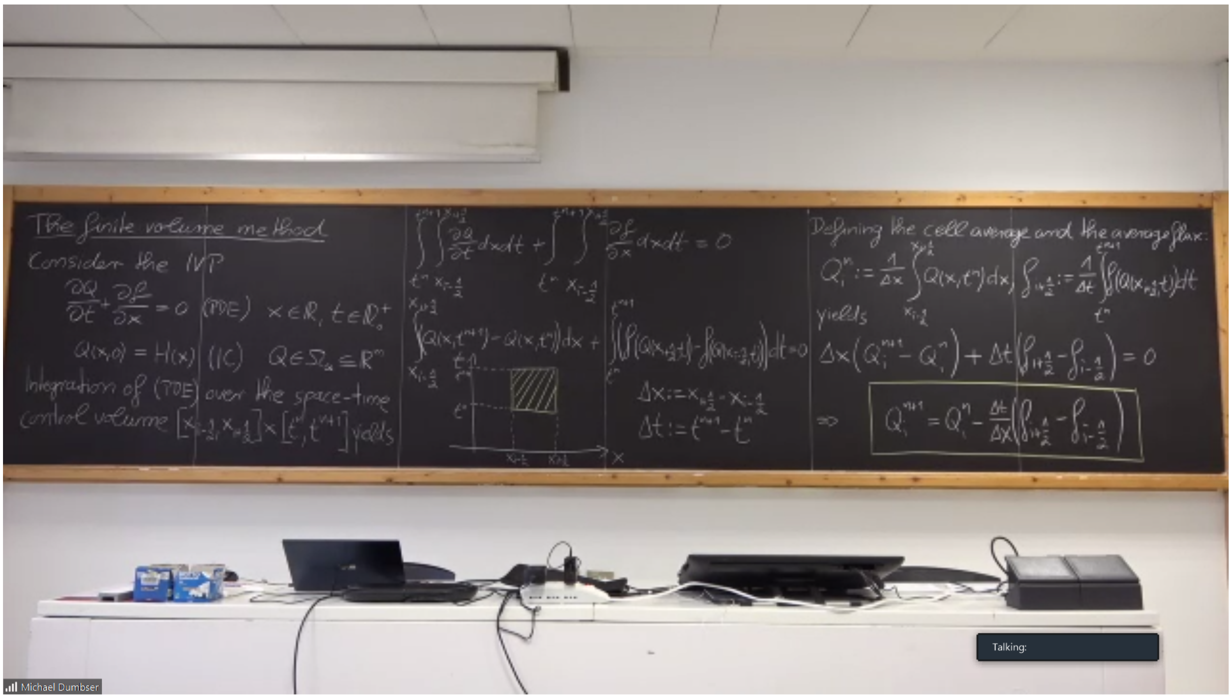

Figure 6: Screenshot of the video content received by a ZOOM client when using the standard ZOOM video feature, which automatically reduces video quality to 360p due to pandemic internet bandwidth restrictions for meetings with more than two participants. The blackboard content is only barely readable.

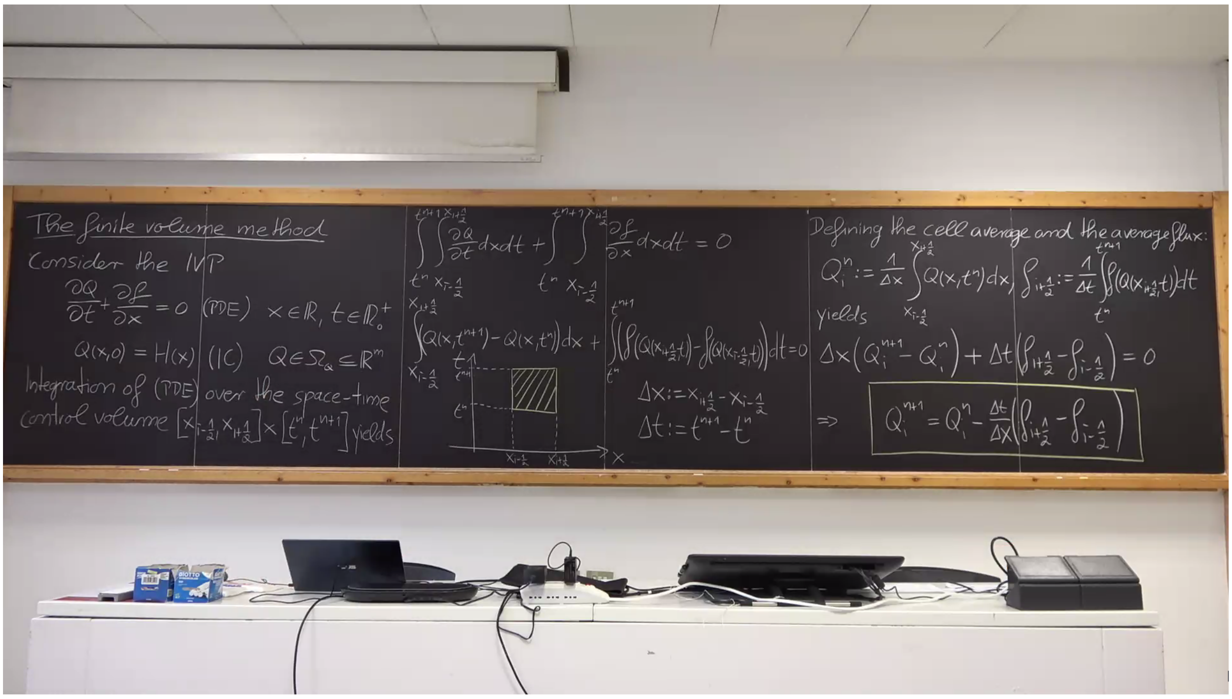

Figure 7: Screenshot of the video content received by a ZOOM client when using the high quality low bandwidth screensharing methodology proposed in this paper at 1080p. The blackboard content is very clearly readable, even including small indices and superscripts.

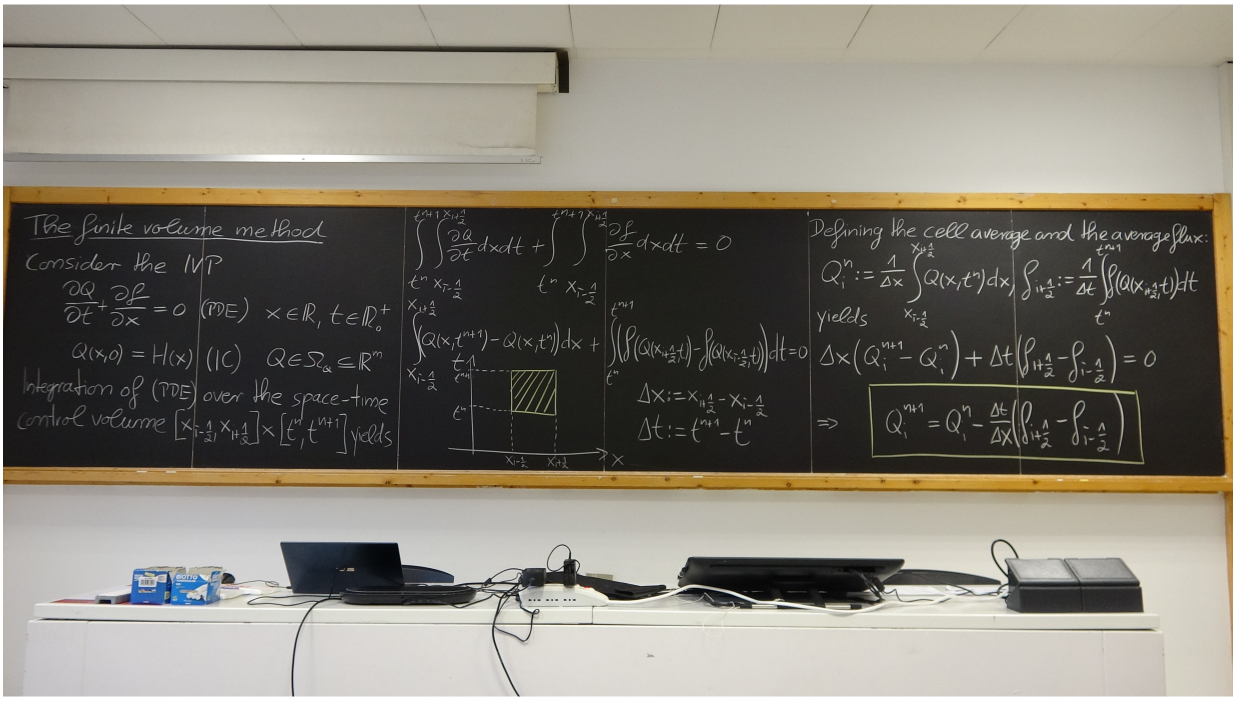

Figure 8: Photograph of the blackboard content taken directly in the lecture hall at 2160p, which serves as a reference for the previous figures.